

Q6

$$f'(x) = \frac{[D(1-e^{-x})] \cdot (1+e^{1+x}) - [D(1+e^{-x})] \cdot (1-e^{-x})}{(1+e^{1-x})^2}$$

$$f'(x) = \frac{2e^{-x}}{(1+e^{1-x})^2} \geq 0 \quad \forall x \in \mathbb{R}$$

$f'(x)$ è MONOTONA CRESCENTE

QUINDI $f(x)$ È INVERTIBILE

$$\left[D f^{-1}(x) \right]_{x_0=0} \quad \begin{array}{l} x_0 = f(y_0) \\ f(y_0) = 0 \end{array} \longrightarrow \frac{1-e^{1-y}}{1+e^{1-y}} = 0 \Rightarrow \boxed{y=1}$$

$$\left[D f^{-1}(x) \right]_{x_0=0} = \frac{1}{Df(x) \Big|_{y=1}} = \frac{(1+e^0)^2}{2 \cdot e^0} = \frac{4}{2} = 2$$