

$$\sin(\alpha + \beta) = \cos\left(\frac{\pi}{2} - (\alpha + \beta)\right) = \cos\left[\left(\frac{\pi}{2} - \alpha\right) - \beta\right]$$

Ex. 89

$$\sin\left(\alpha + \frac{\pi}{4}\right) - \cos\left(\alpha + \frac{\pi}{4}\right) =$$

$$= \sin \alpha \cos \frac{\pi}{4} + \cos \alpha \sin \frac{\pi}{4} - \cos \alpha \cos \frac{\pi}{4} + \sin \alpha \sin \frac{\pi}{4} =$$

$$\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} = \cancel{2} \frac{\sqrt{2}}{2} \sin \alpha$$

$$\cos 2\alpha = \cos(\alpha + \alpha) = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1 = 1 - 2\sin^2 \alpha$$

$$\sin 2\alpha = \sin(\alpha + \alpha) = 2\cos \alpha \sin \alpha$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\beta = 2\alpha$$

$$\alpha = \frac{\beta}{2}$$

$$\cos \beta = 2\cos^2 \frac{\beta}{2} - 1$$

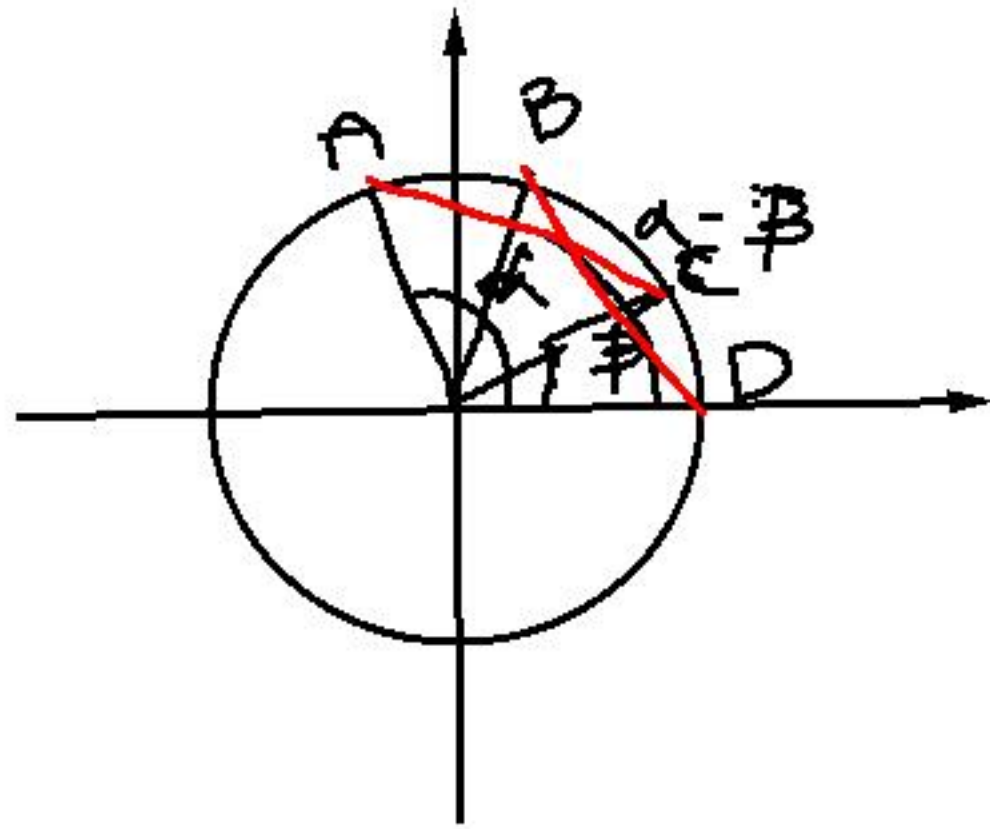
$$\cos^2 \frac{\beta}{2} = \frac{1 + \cos \beta}{2}$$

$$\sin^2 \frac{\beta}{2} = \frac{1 - \cos \beta}{2}$$

$$\cos \frac{\beta}{2} = \pm \sqrt{\frac{1 + \cos \beta}{2}} \quad \begin{matrix} \frac{1}{2}\pi < \frac{\beta}{2} < \frac{3}{2}\pi \\ \frac{1}{2}\pi < \frac{\beta}{2} < \frac{3}{2}\pi \end{matrix}$$

$$\sin \frac{\beta}{2} = \pm \sqrt{\frac{1 - \cos \beta}{2}} \quad \begin{matrix} 0 < \frac{\beta}{2} < \pi \\ \pi < \frac{\beta}{2} < 2\pi \end{matrix}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$



$$\begin{aligned} A &= (\cos \alpha; \sin \alpha) \\ C &= (\cos \alpha; 0) \\ B &= (\cos \beta; \sin \beta) \\ D &= (\cos \beta; 0) \end{aligned}$$

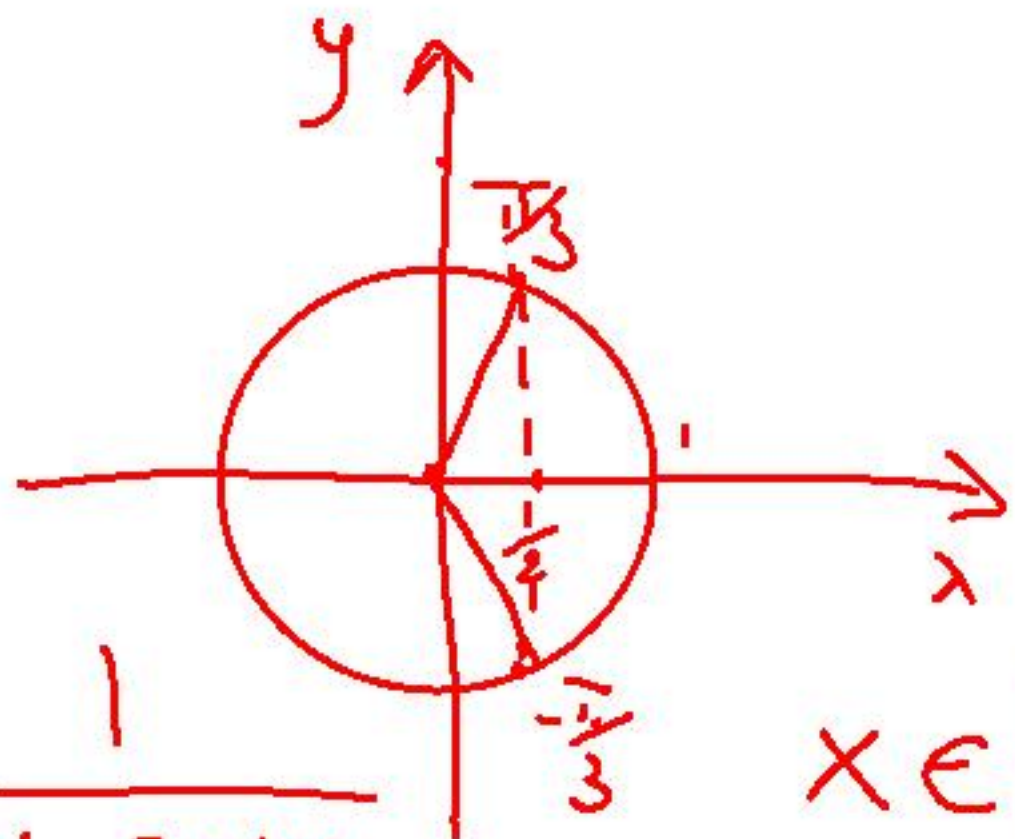
$$\overline{AC}^2 = \overline{BD}^2$$

$$\begin{aligned} (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2 \\ (1 - \cos(\alpha - \beta))^2 + (0 - \sin(\alpha - \beta))^2 \end{aligned}$$

$$D: 1 - 2\cos x \neq 0$$

$$\cos x \neq \frac{1}{2}$$

$$x \neq \pm \frac{\pi}{3}$$



$$f(x) = \frac{1}{1 - 2\cos x}$$

$$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right]$$

MAX?

MIN?

$$\cos \frac{\pi}{2} = 0 \quad -1 \leq \cos x \leq 0$$

$$\cos \frac{3}{2}\pi = 0$$

$$0 \leq -2\cos x \leq 2 \quad \left| \quad \frac{1}{3} \leq \frac{1}{1 - 2\cos x} \leq 1 \right.$$

